## Example : Production scheduling problem

Let us consider the problem having following data :

| Month | Demand | Set up cost | Inventory Cost | Production cost |
|---|---|---|---|---|
| 1 | 80 | 60 | 2 | 5 |
| 2 | 60 | 40 | 2 | 4 |
| 3 | 40 | 60 | 1 | 5 |
| 4 | 70 | 45 | 2 | 5 |

Again, consider a single product and its demand for four periods or four months given as 80, 60, 40 and 70. There is a setup cost that is incurred whenever there is production. For example, if production is to be done in the first month then in addition to the cost of production there is a cost of setup which is say rupees 60. So, the setup costs are given as 60, 40, 60 and 45. There is also a cost of production which is given as 5, 4, 5 and 5. In addition there is an inventory cost which means one can produce in say month one and use the production to meet the demand of month two. In such case a certain inventory cost or inventory holding cost would be incurred. If hundred items have been produced in the first month and consume 80 the remaining 20 is assumed to be carried from the first month to the second month, and it attracts an inventory cost of 2, which is the inventory carrying cost at the end of month one. So, these costs represent the inventory carrying at the end of each month, there is also a production cost which is the cost required or cost that is incurred in producing these quantities. If it is economical and profitable one can produce the total demand is 180 plus 70, 250. So, if it is economical and profitable, one can produce all the 250 in the first month itself there is no limit on the production capacity at the moment. So, there are three costs that are involved the cost of setting up, the cost of production and the cost of holding inventory. In our problem we have also assumed that the setup costs are different in different periods, inventory costs are different in different periods and production costs are also different in different periods. So, the decision or the planning decision is to find out, how much is to be produced in these four months in each of the four months such that the total cost of setup total cost of inventory and production the three cost put together is minimized. So, it is a minimization problem to minimize the cost of setup inventory and production. Time is not taken into consideration. If there is a production in a certain month a certain amount of setup cost is incurred if we decide to produce in this month

The difference comes from the fact that for the first time setup cost is considered. Previous models that have been discussed did not include setup cost explicitly. This model includes setup cost explicitly. A set up cost is said to incurred if production is to be done in a particular month. Every month if there is a production there is going to be a setup. One may also say that this facility can be used to produce more items and products and therefore, if this particular product is made in month one, as well as in month two, then we incur two setup costs one for each month. The problem can be formulated and solved as an integer programming problem with of course, a binary variable that would say that $y_i$ equal to one, if we decide to produce in month i and $y_i$ equal to 0, if we do not produce anything in month i. And constraints that relate the decision to produce which is the $y_i$ and $x_i$ which are the quantities that are produced in this month. This problem does not have an initial inventory there is no product that is available, there is no desired ending inventory. The decision variable will be the quantity that is produced in each period and the criterion of effectiveness or the objective function will be to minimize the sum of the setup costs, inventory costs and production cost. Now, we all know that dynamic programming, though very useful to solve certain classes of problems has its own issues with respect to model. So, if we consider a problem of this type, the state variable can take very large numbers. For example, total demand for the four periods is 250. So, if we use the tabular approach of dynamic programming to solve this, the state variable, which is the amount of inventory available can take very large number of values, and the tables can become lengthy and cumbersome, though dynamic programming can be used to solve this. One of the most interesting simplifications to the dynamic programming solution came from **Wagner and Whitin** and it is called the **Wagner and Whitin algorithm** which uses dynamic programming to solve this particular problem. The Wagner Whitin idea or principle is as follows.

Now there are four periods and we are going to assume that we are going to make decisions on production at the beginning of each of these four periods. Now, it is only logical that if we produce in a certain period, then we will have to meet the demand of that period at least. Otherwise we will have to incur one more setup and one more setup cost, therefore it is uneconomical to consider two setups in a particular period. So, the if there is only going to be one setup in a particular period then it implies that what we produce with what we produce, we should be able to meet the demand of the entire period. One other important assumption is that the demands have to be met at the beginning of each of these periods, shortages and backorders are not allowed in this particular model. So, under that assumption if we produce something in a month, then either the quantity that is produced should be at least the demand of that month or with the beginning inventory and production in that month, we should be able to meet the entire demand of that month. So, if there is a beginning inventory in the beginning of a particular month, let us say we are looking at month number two and let us say if there is a beginning inventory of say 20 in the beginning of month two, then we should produce 40 or more, but then it is also logical to say that if there is a beginning inventory at the beginning of this month, then it should be sufficiently large to meet the demand of this month, then there is no need to setup and produce in this month. So, Wagner and Whitin algorithm works around these ideas. So there are two important points there - one is there will be production in a particular month, when only when the beginning inventory is zero. And if there is production in a particular month then that production quantity will be the demand of that month or the demand of that month and the next month and the demand of three consecutive months or the production quantity is always equal to the demands of certain number of consecutive months. The second if there is beginning inventory in a month, then that inventory should be sufficient to meet the demand of that month and there will be no production in that month. So, the two important results of Wagner and Whitin are if the beginning inventory is 0 or only when the beginning inventory is 0 we will decide to produce in that month. And if we produce, the production quantity is equal to the demand of that month or the demand of two months or demand of three months, or demand of a certain integral number of months. If there is beginning inventory then we will not produce and that beginning inventory should be sufficient to meet the demand of this particular month. The moment we accept and use these two results that came from Wagner and Whitin, the dynamic programming solution to this problem becomes extremely simple. Now, we will look at how to solve this using dynamic programming, under the assumption that backordering is not allowed.

So, let us go back and use the DP solution - dynamic programming solution. So, let us start with trying to meet the demand of month one. So, we first look at trying to meet the demand of month one, there is no beginning inventory so we have to produce in month one. So, demand of month one is met only in one way by deciding to produce. So, we produce the cost equal to production quantity is equal to 80 because demand is 80 and the best way to meet this 80 is given by total cost is equal to setup cost is 60, which is to be incurred production cost is 5 into 80 equal to 460. There is no inventory that is carried because demand is 80, production is also 80, there is only one way to do it which is the optimal way. So, we just put a star indicating that this is the best way to produce and meet the demand of the first month.

Total cost (month1)=60+80*5=460

Now, we go to demand of month two, now the month two demand can be met in two ways one is to try and meet the first month's demand in the most economical way and use the previous result, which is the essence of dynamic programming where the optimal decision up to the previous stage is computed and used for decision making in the next stage. So, one, is to try and use you meet the month one's demand in the best possible way, which is 460 which comes from here plus produce what is required for month two by incurring a setup cost of 40 plus production cost of producing 60 units. We just write TC total cost is equal to - total cost of meeting this second month's demand as well as the first month's demand, two months demand is to meet the first month's demand in the best possible way, which is 460 that comes from here. And then meet the second month's demand alone by incurring a setup cost of 40 and a production cost of 60 into 4 i.e 240. So, this gives a total cost of 240 plus 40 is 280. Hence the cost that is associated with this solution is 460+280 i.e. 740. The other way to meet month one and two demand is to produce everything in the beginning of the first month. So, that would give us Q equal to 140. So, all the 140 units are produced in the first month so there is a setup cost of 60 that is incurred, all the 140 units are produce in the first month. So 140 *5= 700 (production cost) plus inventory cost of carrying 60 units from first month to second month. And therefore, there is an inventory carrying cost =60 *2, this 2 represents the cost of carrying one unit of inventory at the end of the first month to the second month. So, 60 into this 2 is 120 so this becomes 700. The second way of fulfilling the second month demand would incur a total cost of (60+140*5+60*2=880). Now, the demand of months one and two put together can be met in two ways one which gives us a cost of 740, the other that gives us a cost of 880. And since we are trying to minimize 740 is the best way to do this.

Month 2

$cost_1^{(2)}$ = 460+40+60*4=740 **optimal**

$cost_2^{(2)}$ = 60+140*5+60*2=880

Now, we go to month three now. Month three can be met in three ways, one is to meet month one in the best possible way and then produce for two and three in month two.

$cost_1^{(3)}$ =460+40+100*4+40*2=980 **optimal**

The other is to meet month two in the best possible way and produce for month three in month three.

$Cost_2^{(3)}$ =740+60+40*5=1000

And the third is to produce everything in the first month and carry for months two and three.

$Cost_3^{(3)}$ =60+180*5+100*2+40*2=1240

So the optimal cost will be given by first way.

Go to month four: the total cost for month four in four ways:

one is to produce all of these in the first month itself and carry

$cost_1^{(4)}$ =60+250*5+170*2+110*2+70*1=1940

The second is to produce optimally till first month and then produce the rest in second month.

$Cost_2^{(4)}$=460+40+170*4+110*2+70*1=1470

Third is to produce optimally till second month and produce the rest in third month.

$Cost_3^{(4)}$=740+60+110*5+70*1=1420

The fourth is to produce optimally till third month and produce rest in fourth month.

$Cost_4^{(4)}$=980+45+70*5=1375 **optimal**

The optimal solution is obtained in the last case which use optimal solution of third month. Now, go back to this 980 and find out the production quantity which is hundred which comes from 60 plus 40 so Q 2 is hundred. The optimal solution here which happens at this point production quantity is 80 so Q 1 is equal to 80. Now, total cost is equal to 1375. So, we have now solved this optimally using dynamic programming now there are four periods the optimal solution does not ask you to produce in all of the four periods. In some sense it is it is understandable because the production quantity comes down here, so there will be a tendency to push the next month's production into this month, pushing this might involve a higher inventory holding cost. So, it is a tradeoff between inventory holding cost and additional change over or setup cost, but if the production cost themselves change, then this will also have a bearing on the solution. Now, in order to bring out the features of the algorithm we have considered a problem where the inventory holding costs are different in different periods, production costs are different in different periods, and setup costs are also different in different periods. Though one cannot exclude this from happening which means in organizations, we will realize at some point when these costs are measured, one would realize that to setup for the same product the setup cost can vary in different months. The reason could be many sometimes even the labour associated with the setup the cost can change depending on pay revision and additional payments made to the people. And therefore, setup costs can change. Inventory cost can also change due to many reasons for example, if inventory space is added in a certain period or space is removed from the storage area, then there can be a situation where the inventory cost can change. Production cost can also change due to cost of labor and multiple other cause of consumable and so on. So, these things can actually happen, but this model is also applicable when the production costs are the same, the inventory costs are the same and the setup costs are the same across all periods. There it becomes a tradeoff between cost of holding and additional inventory for a particular period versus the cost of incurring one setup or one change over. The point I am making is if all these costs are the same like 60, if all these costs are the same 2, all these costs are the same say 5, then it is a tradeoff between incurring a setup of 60 and carrying a certain quantity of the demand for the next period multiplying it by two. Wherever it is cheaper than we would chose to setup or we would chose to produce early, and carry the inventory. So, the Wagner Whitin algorithm actually has become, once we apply a Wagner Whitin idea into the dynamic programming.

Model: